Modern Surveying Technology

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Modern technology, especially in electronics, has come to the surveying industry. At two recent ACSM/ASPRS conventions, the number of exhibits promoting computers and computer software may have easily outnumbered the surveying instrument and supply dealers, but the surveying manufacturers have also joined the electronics revolution. Electronic Distance Meters have been a standard accessory for surveying firms for many years, and the electronic theodolite is now a standard item for every major manufacturer.

One of the problems with theodolites, whether electronic or optical, is that you can't be sure of the instrument's precision. When the engineer's transit was the standard of the industry, precision was determined by the least count of the vernier. With theodolites, whether scale-reading, micrometer or electronic, the least count of the reading or display system means very little. An instrument with a least count of one second of arc (1") may have a precision of 3"-4", and an instrument with a 10" least count may have a precision of 2"-3". The following will describe how you can determine a theodolite's precision.

By precision we mean the ability to repeat itself. All theodolites and transits are basically accurate because the horizontal and vertical circles close on themselves, but precision is a measure of how well the circles were graduated, and how well the entire theodolite is constructed.

For this test, select an observation station at a location where two well-defined points are visible at least 500' away (a pair of traverse targets or natural objects). The angular separation of these targets is not too critical, but angles near 90° seem to be ideal.

1. With the instrument carefully levelled and positioned directly over the occupied point, in the direct position (vertical circle on the left side as you look through the telescope), set the horizontal circle to zero and find point the left target, using the lower clamp and drive if the instrument is a repeating theodolite. With a direction theodolite, fine point the target, then drive the circle to zero.

2. Slacken the upper plate clamp. Turn to right target. Tighten plate clamp. Fine point with plate drive. Read and record horizontal circle reading.

3. Invert (plunge) the telescope. This is the reverse position. Slacken the upper plate clamp. Turn back to right target. Tighten plate clamp. Fine point with plate drive. Read and record horizontal circle reading.

4. Slacken the upper plate clamp. Turn to left target. Tighten clamp. Fine point with plate drive. Read and record horizontal circle reading.

This is a set of observations. There should be at least 8 sets observed in order to get a decent measure of precision.

Table 1 contains 8 sets of data observed with a 6" micrometer theodolite, using the method described above. Each column is numbered at the top of the table.

The numbers before column 3 indicate the order in which the observations (column 4) were taken. The initial reading, in set 1 to station 1 in the direct position, was "0°00'0" or "zero". For the additional sets, the horizontal circle was rotated so that the initial circle reading on the left target is no longer zero, and the observations are being measured on a different part of the circle. A rule of thumb to use is, if "n" sets are to be observed, the circle is rotated by 180°/n for each set. Also, change the minutes and seconds if using a micrometer theodolite.

In this example, 180°/8=22 1/2°. You do not have to be a purist and advance the circle exactly 22 1/2° since ballpark values are "OK" for a test.

Column 5 is the mean of the two observations in column 4. The "degrees" and "minutes" in column 5 are the same as the first value of the two readings in column 4. The mean is really the mean of the "seconds".

FIGURE 1

1 Set	2 Station		3 D/R	4 Circle Reading	5 Mean	6 Angle
1	1	1	D	0-00-00	0-00-15	105-01-03
		4	R	180-00-30		
	2	2	D	105-01-12	105-01-18	
		3	R	285-01-24		
2	1	8	D	21-52-24	21-52-27	105-00-51
		5	R	201-52-30		
	2	7	D	126-53-12	126-53-18	
		6	R	306-53-24		
3	1	9	D	45-38-06	45-38-15	105-00-45
		12	R	225-38-24		
	2	10	D	150-38-54	150-39-00	
		11	R	330-39-06		
4	1	16	D	67-32-12	67-32-12	105-00-57
		13	R	247-32-12		
	2	15	D	172-33-06	172-33-06	
		14	R	352-33-12		
5	1	17	D	90-25-24	90-25-30	105-00-57
		20	R	270-25-36		
	2	18	D	195-26-18	195-26-27	
		19	R	15-26-36		
6	1	24	D	112-25-12	112-25-21	105-01-00
		21	R	292-25-30		
	2	23	D	217-26-12	217-26-21	
		22	R	37-26-30		
7	1	25	D	145-46-54	145-47-03	150-01-00
		28	R	325-47-12		
	2	26	D	250-47-54	250-48-03	
		27	R	70-48-12		
8	1	32	D	167-56-12	167-56-21	105-01-00
		29	R	347-56-30		
	2	31	D	272-57-18	272-57-21	
		30	R	92-57-24		

FIGURE 2

1 Angle	2 v	3 v^2
105° 01' 03"	+ 6.37	40.5769
105° 00' 51"	- 5.63	31.6969
105° 00' 45"	- 11.63	135.2569
105° 00' 57"	+ 0.37	0.1369
105° 00' 57"	+ 0.37	0.1369
105° 01' 00"	+ 3.37	11.3569
105° 01' 00"	+ 3.37	11.3569
105° 01' 00"	+ 3.37	11.3569
Mean= 105° 00' 56.63"	-0.04	$\Sigma v^2 = 241.8752$

$$\sigma_s = \sqrt{\frac{\Sigma v^2}{n-1}} = \sqrt{\frac{241.8752}{7}} \qquad \sigma_s = 5.9$$

Column 6 is the *angle* that was observed in each set. It is determined by subtracting the mean of the readings to station 1 from the mean of the readings to station 2 (column 5).

The data from Table 1 gives you the numbers necessary to determine the precision of the theodolite. See Table 2.

Column 1 contains the eight angles determined in Table 1. At the bottom of column 1 is the mean of these eight values. The value of the mean is carried to two more digits after the decimal point than the angles.

Column 2 has the title "*v*", which is the international symbol for "residuals." A residual is the quantity "observation minus the mean." As an example, the first residual is

$$\begin{array}{r} 105°01'03.00'' \\ \underline{-105°00'56.63''} \\ +\ 6.37'' \end{array}$$

Record the sign (+ or -) with each value. Theoretically, the sum of the residuals should be "zero," but because of roundoff when determining the mean, it may be a very small number. If this number is anything other than a very small number, one of the residuals is wrong.

Column 3 is "v^2," the square of the residuals. Here, all numbers are positive. The values are carried out to 4 digits after the decimal point, which is what you would read on your calculator. The sum of all the v^2 in the column is Σv^2.

You can now compute the "standard deviation"-the number that describes the precision of the instrument. Standard deviation is denoted by the Greek symbol σ_s. The equation for standard deviation is:

$$\sigma_s = \pm\sqrt{\frac{\Sigma v^2}{(n-1)}}$$

Σv^2=the summation of column 3
n=number of values used in the calculation (8 in this example).
Using the values from Table 1,

$$\sigma_s = \pm\ 5.9''$$

"How does this number tell me the precision of my theodolite? The value σ_s= 5.9" seconds of arc is the standard deviation of a single observation. However, eachof the eight values of angles given in Table 1 is the meanof a direct and reverse reading. Therefore, σ_s is the standard deviation of the mean of a measurement taken in the direct and reverse positions. This is what instrument manufacturers use to describe the precision of their instruments. In this example, the least count of the instrument was 6" and σ_s = 5.9". But don't be shocked if your theodolite has a value of σ_s much higher than the last count.

Another statistic available from the data in Table 2 that is used by surveyors who observe more than 1 set of data at each station is standard deviation of the mean.

The equation for σ_m is

$$\sigma_m = \pm\sqrt{\frac{\Sigma v^2}{n(n-1)}} = \frac{\sigma_s}{\sqrt{n}}$$

In this example, $\sigma_m = \pm\ 2.1''$

This shows that the mean of a series of observations is always a better value than any single observation. We have determined that the standard deviation of the mean of a single observation taken in both the direct and reverse positions, for this particular theodolite, is 5.9". However, by repeating this series of observations eight times, the standard deviation of the mean is lowered to 2.1".